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A Survey of Some Recent Developments in Macroeconomics

by

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ABSTRACT

This paper reviews some current issues in dynamic macroeconomics, focusing on the common framework of dynamic general equilibrium theory as applied to many different problems. The issues reviewed are: a) Dynamic Programming, b) Real Business Cycle Theory, c) Endogeneous Growth Theory and d) Policy Issues in Dynamic Models.

RÉSUMÉ

Le présent document traite de différents aspects concernant la macroéconomie dynamique en se concentrant sur le cadre d'analyse de la théorie d'équilibre générale dynamique. Plus particulièrement, les sujets étudiés sont: a) la programmation dynamique, b) la théorie des cycles réels, c) la théorie de la croissance endogène et d) divers aspects politiques des modèles dynamiques.

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INTRODUCTION

Macroeconomic theory has gone through a number of major changes in the last two decades, changes which involve conceptual structure, the range of issues addressed, methodology, and technique.[1] A student transported from the late 1960's or even early 70's would find much of the material on a typical macroeconomics graduate course quite unrecognizable. The use of rational expectations, the application of fundamental microeconomic principles of behaviour, the development of a wide range of techniques in dynamic and stochastic analysis, the adoption of strategic concepts from game theory; these and other elements have significantly altered the way macroeconomic theory is done today relative to fifteen or twenty years ago.

What precipitated these dramatic changes in the discipline? Identifying causal influences is difficult. One of the major criticisms made of the the conventional textbook macroeconomics has been its tenuous link with basic economic theory - its lack of 'microeconomic foundations'. Certainly one of the major motivations behind the recent literature has been to provide improved behavioral explanations of macroeconomic phenomena. It can be said now with some justification that the gulf between microeconomic and macroeconomic theory has been bridged in many places.

More important however, has been the lack of confidence in the empirical results of the conventional macro theory. In the 1960's and early 1970's there appeared to be widespread consensus on the applicability of large-scale macro-models for evaluating the effects of activist monetary and fiscal policies. This consensus largely broke down in the late 1970's and 80's. The popular rational-expectations/monetary misperceptions models also performed very poorly at the empirical level. While macroeconometric models

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[1] For some recent surveys, see Fischer (1988) and Lucas (1987).

are still useful for short-term forecasting and are still widely used for this purpose, there are few economists who would recommend the use of activist stabilization policies from a 'menu' provided by these models with the same luster of twenty years ago.

A related issue is the change in the policy agenda that has taken place. Many of the macroeconomic issues that are pressing in current times, - budget deficits, real exchange rate volatility, international debt, oil prices, etc, were not even part of the debate of two decades ago. New tools and new perspectives were often perceived as necessary to deal with these new issues as they arose. It is possible to trace a lot of the theoretical work in macroeconomics back to particular macroeconomic episodes.

We should also note the influence of the vast increase in technical abilities of economists, both at a purely analytical level and from the use of computers. Many problems that are analyzed now, both theoretical and applied, could simply not have been addressed in a rigorous fashion a decade ago. A particularly relevant example is the use of neo-classical growth theory to address issues relating to business cycles - the 'real business cycle' theory discussed below. At the time that these growth models were developed, in the 1960's, it could not have been envisaged how relevant they would be for the set of problems they are used to address today. To a great extent, the technologies of the day must dictate the current research agenda.

This paper reviews some recent developments in macroeconomic theory. Rather than giving a chronological account of the literature or presenting an exhaustive taxonomy of models, we select a number of topics of current interest and analyze them in some depth. While there is a bewildering variety of models in usage among the profession, in many cases there is cases a common underlying structure to the models. In this introduction we discuss

some themes that occur repeatedly throughout the paper.

The task of macroeconomic theory is to develop internally consistent conceptual frameworks, grounded in basic economic theory, which can be used to understand observed aggregate economic data, and can answer questions about the effects of government policy. One of the principal developments in recent years has been the use of intertemporal stochastic general equilibrium models. A general equilibrium model roughly speaking represents a conceptual structure in which the objectives of all the agents populating the economy are fully spelled out, the budget or balance sheet constraints which they face are stated explicitly, as well as the technology of production available to them, and the market structure, or in other terms the way in which an equilibrium occurs, is defined. Thus a general equilibrium model is "complete" and internally consistent in the sense that it may be used as a "model economy". Given its assumptions about what we call "primitives": objectives and constraints, it contains predictions about endogenous market outcomes. It can give positive answers to questions of interest to a macroeconomist.

The belief in the explanatory power of general equilibrium models may be traced back to the work of Robert Lucas at the University of Chicago during the 1970's. Lucas was one of the main forces behind the development of "rational expectations" modeling.[2] This is the idea that people's forecasts of prices and quantities are not arbitrary numbers but are related in some way statistically to the actual economic mechanisms which determine these prices and quantities. Thus in developing a macroeconomic model where expectations are important we should determine expectations <u>within the model</u>.

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[2] See, for instance, Lucas and Sargent, ed. (1980).

The wider implication of this was argued in a paper by Lucas in 1976. Starting from the premise that economic agents behave purposefully in response to information available to them, as well as their immediate budget constraints, we must infer that our observations as macroeconomists must represent some aggregation of rational individual choices. In order to evaluate the impact of government policy, we have to know how the underlying decisions of agents will respond. In other words, policy not only directly impacts on the economy but also can indirectly change the economy's observed structure through this response. If we are to adequately investigate the effects of policy changes we must know something about individual economic response to changes in constraints; in other words we have to model the decisions of individuals populating our economy. The assumption that they behave according to a fixed pattern in response to changes in their environment is likely to be mistaken and may give seriously wrong answers to policy questions. To give complete answers to these questions we need complete models, i.e. general equilibrium models. These types of models can predict how agents will respond to arbitrary changes in their environment, and how this will affect macroeconomic outcomes.

Most of the recent vintage of macroeconomic models are dynamic, meaning that the decision makers in these economies are making decisions over time. Many, if not all interesting macroeconomic problems involve the passage of time, e.g. investment, budget deficits, inflation, business cycles, etc. A static view of the macroeconomic equilibrium as being determined by atemporal supply and demand curves is thus not likely to get us very far in understanding these processes. The analytics of intertemporal decision making have been developed in many directions in recent years, as part of the "dynamicization" of macroeconomic theory. Section I below describes the

operation of these tools in some detail.

The important and complicating feature of dynamic general equilibrium models is that in choosing an action the decision-maker has to take the future into account. Thus all choices implicitly involve contingency plans for the future. The decision to consume more today is simultaneously a decision to consume less in the future for a consumer who is constrained by an intertemporal budget constraint. One of the strong implications of this logic is that government policy cannot be thought of as a single action, but as a sequence of actions spread out over time, all of which must in some way be consistent with one another. Policy has to be thought of as a "regime" rather than an event. How does an increase in the corporate tax rate affect private investment? In the environment which we are describing, this question cannot be answered unless we know or can make a forecast of a) the future movement in the tax, and b) the distribution of revenues from the tax. The general equilibrium methodology imposes a powerful discipline on the researcher, forcing him to rigorously specify the experiment conducted. In section III below we describe some unusual answers to correctly specified policy experiments.

Many of the models used in the field are stochastic, reflecting the fact that decision-making under uncertainty is a central feature of all economies. (Decision-making under uncertainty is discussed in Section I.) Uncertainty is introduced into these models by making explicit assumptions about the probability distribution of shocks hitting the economy, such as technology shocks, terms of trade shocks, etc. The combination of an uncertain and dynamic environment are in many instances critical to the ability of these models to provide coherent explanations for economic phenomena. An important case is that of business cycle theory, treated in detail in Section II below.

In these models, exogenous shocks hit the model economy according to some random process, but it is the dynamic response of individuals to these shocks which leads to their "propagation" through time, giving rise to business-cycle like phenomena.

It should be noted that there is a critical difference between introducing uncertainty at the level of individual decision-making in the model, and exploring its consequences for aggregate outcomes, on the one hand, and adding on an "error-term" to estimating equations in an econometric model, on the other. The first procedure involves the economics of uncertainty, while the second merely satisfies a statistical requirement.

An important feature of general equilibrium models is that in principle they attempt to explain "everything". By this we mean that they are self-contained microcosms of an aggregate economy and depending on how sophisticated the model is, make predictions for many features of the economy simultaneously. Thus an aggregative general equilibrium model of investment behaviour must also contain predictions for behaviour of rates of return, output, consumption, productivity, etc. This is problematic to the extent that all models must necessarily be highly simplified and abstract, and some of its predictions may seriously be in conflict with the evidence. On the other hand, an intellectual discipline is imposed on the researcher, forcing him to develop models in which all features square with the empirical evidence. This is precisely the method of real business cycle research, discussed in Section II.

Finally, a word must be said about the role of policy in the overall framework of the new macroeconomic theories. Many of the models presented below cannot give adequate answers to the question of whether government policy is useful or desirable. In fact many implicitly contain an "invisible

hand" feature which establishes that the unfettered market economy is efficient. This should not be taken as a statement of the model-builders belief, but rather as an indication of the early stage of development of these types of models. The analysis of "distorted" economies, in which the role of government is taken seriously, is an outstanding issue in the research agenda on general equilibrium models. We will have some more to say about this in Section IV.

SECTION I : DYNAMIC PROGRAMMING TECHNIQUES

This section attempts to provide, in as intuitive a fashion as possible, a guide to the main techniques employed in the recent vintage of dynamic macroeconomic models.[3] As is clear from the introduction, optimal decision-making over time is central to these models. Mathematically, this leads us to seek solutions to dynamic optimization problems. In other words, we seek a sequence of actions to be taken now and in the future which maximize some objective, such as profits in each year, subject to constraints such as technology and factor prices in each year.

The solution gives us both a maximized present value or expected present value of profits, as well as a sequence of actions or contingency plans for optimal choice of inputs over time.

What is the best way to go about solving such a problem? One way at least would be to take a very Marshallian view of the world, and choose all actions at once for the whole horizon subject to all constraints. This

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[3] There are a number of good references for this section, including Sargent (1987), Harris (1987), and Lucas, Prescott and Stokey (1988). King (1987) provides a good introductory survey of techniques.

essentially reduces the problem to a standard form of maximization subject to constraints familiar from micro texts.

The problem with this procedure is that it is not very revealing. The inevitable answer we get in a problem such as that posed above is that "everything depends upon everything else", which does not get us very far. An alternative line of attack might be designed in the hope that the problem could be solved sequentially. Is there any sense in which we can model the decision-maker as acting period by period, doing the best for this year with an eye to the future, and yet end up with an optimal sequence of actions for the whole horizon? The method of <u>dynamic programming</u> is a technique which does just this. Roughly speaking, it is a technique which breaks down one big maximization problem into many small ones, and in the process, makes the model-builder's job a lot easier.

As in most things, we don't get something for nothing, and the cost in this case is that the type of dynamic decision problem that satisfies the conditions for this sequential separation is a small subset of all the possible ones we could imagine. However, as it turns out, many of the interesting problems in dynamic economic theory can be posed in a fashion for which the method of dynamic programming is especially useful.

We discuss these issues very loosely here. An appendix provides some more rigorous arguments. First of all, take the problem facing a hypothetical economic agent who lives for T years. The agent faces the following problem

(I-1) Maximize $\sum_{t=1}^{T} \beta^{t-1} r_t(X_t, U_t) + T(X_{T+1})$

subject to

$$X_{t+1} = f(X_t, U_t), \quad t = 0, 1, \ldots T, \quad X \text{ fixed}, \quad 0 < \beta < 1.$$

The function $r_t(X_t, U_t)$ is the agent's objective, which might be utility, profits or something else. At each period t, for given X_t and U_t, the agent gets $r_t(X_t, U_t)$ reward, and at the beginning of the planning horizon the total current value reward is given by $\sum_{t=0}^{T} \beta^t r_t + T$. This indicates that rewards in the more distant future $t > 1$, are given less weight then current rewards; i.e. $\beta^t < 1$. The future is discounted. Rewards depend upon two variables. X_t is a state variable. It describes, for each period t, the environment within which the person is placed. For instance, X_t could be a fixed capital stock of a firm or the fixed level of wealth of a saver. $T(X_{T+1})$ is a "scrap value" function - it assigns a value to the end of period state variable. The variable U_t is described as a control. It is chosen at each time period in some optimal manner given the fixed level X_t. For instance U_t could be the amount of investment or labour hired by the firm, or the level of consumptions chosen by a consumer. The choice of U_t, given X_t will affect the evolution of the state variable over time however. This process is described by the sequence of functions $X_{t+1} = g(X_t, U_t)$. This function says that the dynamics of the state variable are determined by the choice of controls. For instance, the movement in a firm's stock of capital will depend upon its investment decisions over time.

The choice facing the agent is now clear. Choose the sequence of controls U_t, constrained by the sequence of equations $X_{t+1} = g(X_t, U_t)$ in order to attain the highest possible value of $\sum_{t=1}^{T} \beta^t r_t$. In principle, this could be done by choosing everything all at once - the typical static microeconomic perspective, mentioned above. However, there is an easier way to go about it. Notice that the way the problem is set up, as the agent moves through time all previous choices of U_t have no relevance for current

or future actions except in so far as they affect the contemporary value of X_t. For instance, imagine the person is at some time $\tau > 0$. Then the problem which remains to solve is

(I-2) Maximize $\sum_{t=\tau}^{T} \beta^{t-\tau} r_t(X_t, U_t) + T(X_{T-1})$

subject to

$$X_{t+1} = g(X_t, U_t) \quad t = \tau \ldots T \quad X_\tau \text{ fixed}$$

Thus the past only matters through its affect on the current state variable X_τ, which is now historically determined from the point of view of the individual. The firms optimal investment decision, to take an example, will depend only on its current capital stock, (which reflects its previous investment decisions), and not independently on its previous investment decisions. This suggests to us that we may be able to sequentially order the decisions in the manner conjectured above.

The argument proceeds as follows. Imagine we are at any time τ. Then the problem facing the decision-maker is to choose the controls for the remaining time $\tau, \tau+1, \ldots T$, subject to the dynamic constraints $X_{t+1} = g(X_t, U_t) \quad t = \tau, \tau+1, \ldots T$, and to the historically given X_τ. Now if this is solved, then we can define the function $W_\tau^*(X_\tau)$ where

$$W_\tau^*(X_\tau) = \sum_{t=\tau}^{T} \beta^{t-\tau} r(X_t^*, U_t^*)$$

where the stars denote the fact that U_t^* and X_t^* have been chosen to maximize the agents objectives. Because of the recursive structure of the dynamic constraints; X_{t+1} depends upon X_t which depends upon X_{t-1}, etc., the only exogenous data in this optimal value is X_τ.

Now let's take the argument back one time period $\tau - 1$. Knowing that whatever value for X_τ is thrown up for next year, the agent will choose an optimal sequence of U_t's to attain $W_\tau(X_\tau^*)$. We can describe the problem

facing the agent at date $\tau - 1$ as

(I-3) Maximize $\left\{ r\ (X_{\tau-1},\ U_{\tau-1}) + \beta\ W^{*}_{\tau}\ (X_{\tau}) \right\}$

subject to $X_{\tau} = g(X_{\tau-1},\ U_{\tau-1})$

Thus the agent at time τ-1 has only the problem of choosing the value of $U_{\tau-1}$, taking into account the value of X_{τ} this will imply, in the knowledge that optimal actions will be taken in the future so that whatever X_{τ} is the welfare value of $W^{*}_{\tau}\ (X_{\tau})$ will be attained. But the solution to the problem defined here must clearly give the maximum welfare at time τ-1; i.e.

(I-4) $W^{*}_{\tau-1}\ (X_{\tau-1})$ = Maximize $\left\{ r\ (X_{\tau-1},\ U_{\tau-1}) + \beta\ W^{*}_{\tau}(X_{\tau}) \right\}$

subject to $X_{\tau} = g(X_{\tau-1},\ U_{\tau-1})$

The basis of dynamic programming is the result, that if we use the correct functions for $W^{*}_{T-\tau}$, then choosing the U_t's to solve the sequence of problems given by (I-4) for each T gives the solutions to the overall maximization problem defined by (I-1). This result comes from what is known as the principle of optimality, which states that in certain maximization problems, including the ones discussed here, a sequence of plans chosen optimally over an interval of time remain optimal as we move forward into the future. Thus, if the firm chooses a series of investment plans to carry out in the future, and these lead to accumulation of capital stock, then as time unfolds it will remain optimal for the firm, given the capital stock in place at any date, to maintain its original investment plans for that date. This means that the intertemporal profit maximization decision may in fact be modeled as a sequence of smaller decisions such as choosing investment at any date subject to the existing capital stock at that date.

This is only a useful exercise if we can say something definite about the W^{*} functions. Otherwise we have just stated an identity without meaning.

It turns out that there are powerful mathematical tools which can be applied to the functional equation (I-4) which can give us exactly the form of the W^* function. The appendix describes a number of solution techniques.

A solution to this sequential problem gives us not only values of W^* for each period, but more importantly, gives us what may be described as optimal policy rules $U_t = h_t(X_t)$ (where policy there does not necessarily mean government policy) for each time period t, relating optimal controls to the state. This is an extremely useful result, because it actually characterizes the structure of decision-making in real time in a way that general solutions to the original large maximization problem does not. In terms of empirical economics, knowing the function $h_t(X_t)$ gives us valuable information about the response of agents to changes in their environment. Thus to answer policy questions that get around the "Lucas critique", we need to know something about $h_t(X_t)$. Since the solution for the control variable is obtained, we also have a solution for the dynamics of the state variable X_t, over the planning horizon; re. e.g. knowing the firms investment decisions, we know the evolution of the capital stock over time.

Now both the value function solution W^* as well as the optimal policy functions h_t will explicitly depend upon time; i.e. the h policy for $t=\tau$ differs from that of $\tau+1$. This is an undesirable feature. We would like to have time invariant decision rules; i.e. a rule h which says that for a given X_t, U_t will be given independent of the particular year that we are in. This property is essential if we are to match our results with time series data.

The only reason that h and W depend upon time is that the planning horizon faced by the agent is finite. Thus, where we are in the finite interval makes a difference. The common technique used to derive time invariant solutions is to let $T \rightarrow \infty$; i.e. assume an infinite horizon. Then

the policy function h and the value function W become time invariant; i.e. for any X_τ, their values are independent of t. This assumption is not as bizarre as it may seem at first sight. In many decision problems there is no natural end period; e.g. does IBM plan over a horizon which terminates after some date T? If the termination date is uncertain and far in the future then an infinite horizon is generally a good approximation.

With time invariant decision rules $U_t = h(X_t)$ for any ordinary t we have generated strong testable predictions about the response of agents to their economic situation, captured by the state variable X_t. If we have specified the objectives r and technology g correctly, these decision rules should characterize any economy; i.e. they are fundamental predictions of dynamic economic theory in a sense in which arbitrary consumption or investment functions are not.

Finally with an infinite planning horizon we might think of X_t settling down after a large number of periods so that it approaches a constant number every period. Since with the solution for U_t as $U_t = h(X_t)$ we have

$$X_{t+1} = g(X_t\ h(X_t)) = \tilde{g}(X_t)$$

This is a first order difference equation. If X_t converges after a large number of years, we would expect to approach some value $\tilde{X}$ such that

$$\tilde{X} = \tilde{g}(\tilde{X})$$

This value of $\tilde{X}$ is called the steady state. Thus for whatever problem we are concentrating on, our theory can in principle describe both the dynamics out of steady state and also the determinants of the long run equilibrium.

Before going on to some examples and the relationship between the technique described here and the operation of decentralized market economies, we must sketch out how the procedure is amended in the presence of uncertainty. In order to maintain the extreme convenience of sequential

decision-making in an uncertain environment, we have to introduce uncertainty into the model is a very specific way. Assume now that instead of the dynamic equation in (I-1), we have

$$X_{t+1} = g(X_t, U_t, Z_t)$$

Here Z_t is interpreted as some type of random shock to the constraint the agent is forced with. In the case of a profit maximizing firm this could be a shock to the firm's technology or exogenous cost factors. The Z_t's are random variables for each time t. We assume that for each t there is a finite number of possible values that Z_t can take on, given by $Z_t = (Z_t^1 \ldots Z_t^s)$ Each Z^i occurs with probability θ^i, and these probabilities are the same every period [i.i.d.]. The decision problem facing the agent is now

(I-5) Maximize $E \sum_{t=1}^{\infty} \beta^{t-1} r(X_t, U_t)$

subject to $X_{t+1} = g(X_t, U_t, Z_t)$

The notation E means the expectations operator, which means that the agent must now operate by choosing the highest <u>expected</u> present value of the stream of returns. The only uncertainty here is in the value of the Z terms, so clearly this expectations operator must be related to the distribution of Z.

Now let us assume that uncertainty is resolved in the following manner. A time t the actual value of Z_t is known to the agent involved, but all future Z's Z_{t+1}, Z_{t+2}, etc. are unknown. Then we can reasonably describe the <u>state</u> the agent is faced with in time t as the pair of values X_t, Z_t. Now in the same manner as before, we may describe a <u>value function</u> which gives the agents maximal welfare as a function of the current state by way of the following recursive equation

(I-6) $V(X_t, Z_t) = \text{Maximize}\{r(X, U_t) + \beta E\, V(X_{t+1}, Z_{t+1})\}$

subject to $X_{t+1} = g(X_t, U_t, Z_t)$

The intuition behind this equation is similar to the previous case. The only

relevant data for an optimal choice at any time t is the state (X_t, Z_t). Thus whatever the value that Z_{t+1} takes on, an optimal choice ensures the value $V(X_{t+1}, Z_{t+1})$. The current maximum value function $V(X_t, Z_t)$ is then given by an optimal choice of U_t given the current state and the distribution of all possible future values for Z_{t+1}. Using the assumed distribution for Z, we may write (I-6) as

$$V(X_t, Z_t) = \text{Max } \{r(X_t, U_t) + \beta \sum_{s=1}^{s} \theta_s V(k_{t+1}, Z^s)\}$$

$$\text{subject to } X_{t+1} = g(X_t, U_t, Z_t)$$

Again there exist mathematical results to ensure that (i) the V function is well defined and (ii) there exist optimal policy functions given by

(I-7) $U_t = h(X_t, Z_t)$

which solve the original maximization problem (I-5). These functions are even more useful than those for the case of certainty, because they allow for randomness in the observed decision rules of economic agents, and moreover tie in this randomness to the type of stochastic setting faced by the agent.

Although we are still working in a fairly abstract environment, the solution (I-6) together with the dynamic equation (I-7) can give us some insight into the way in which fluctuations are generated in a competitive economy. The economy (taken now to mean the constraints facing the decision-maker) is subjected to random shocks, technological or otherwise. The way these shocks are <u>propagated</u> through time (i.e. the way they affect successive values of X_t) depend upon the endogenous responses of agents in the economy through the h functions. Thus in principle, we should be able to construct models along these lines which can explain the dynamics of business cycles in competitive economies. This is exactly the subject of the next section.

The model we describe here does not have a steady state in the sense described in the previous environment without uncertainty. This is obvious, as the system will be continually hit by the disturbance Z, so that X cannot settle down to a fixed number over a large period. However, in most cases it can be established that the state will settle down to a steady state distribution function. This means that, asymptotically, the capital stock will converge to a given set in which it will take on a range of values with known, time invariant probabilities.

We now discuss a particular example which fulfills four purposes. It illustrates the use of dynamic programming. It highlights the notion of an intertemporal general equilibrium model. It serves as the first half of the comparison between the social planning problem and the competitive equilibrium problem discussed below in this section, and finally it gives an introduction to the "real business cycle" models discussed in the next section.

This example is borrowed from the literature on optimal growth theory which was developed in the late 1960's. Take a social planner facing the problem

(I-8) Maximize $\sum_{t=0}^{\infty} \beta^t \nu(C_t)$

subject to $C_t + k_{t+1} \leq f(k_t) \quad f' > 0, \; f'' < 0$

$k_o = \bar{k}_o$

Here C_t is consumption, $f(k_t)$ is total output, and k_t is the capital stock. At any time period the planner must divide total output between consumption C_t, and investment in next years capital stock k_{t+1}. The planner places some valuation on consumption in each period, given by the function $\nu(C_t)$. The intertemporal choice problem here is to determine an optimal profile of consumption rates through time; consumption is the single good. To do this

however requires determining the correct mix of output between savings and investment at each time period.

This problem can be posed in a dynamic programming framework, and, provided the functions ν and f satisfy certain conditions, has a solution for the value function and decision rules which are unique. Let us briefly describe how to go about this. Let k_t be the state variable, and k_{t+1} the control. Thus the functional equation analogous to (I-4) is

(I-9) $V(k) = \text{Max } \{U(f(k) - k') + \beta V(k')\}$

Thus the problem becomes one of choosing the future capital stock k' at each period, knowing the value function V and given a level of k predetermined. The solution to this problem gives a policy function

(I-10) $k_{t+1} = h(k_t)$

where h() is a nondecreasing function; i.e. for $k_1 > k_2$ $h(k_1) \geq h(k_2)$ must hold.

Given the policy solution (I-10) we have determined the endogenous dynamics for a general equilibrium economy. This is a complete, general equilibrium model of an economy, albeit very rudimentary, in which the macroeconomic identity Y = C + I holds at every period. While the model makes grossly simplifying assumptions, it was the advantage that it can make strong falsifiable predictions while imposing very little structure other than the existence of preferences and technology. For any initial k_o, we can use this model to get a time series for consumption, investment and output. The model makes a series of predictions which are in principle testable using data from any economy.

It may be revealing to give a particular case for which the dynamics can be worked out exactly. Let $U(C) = \log C$ and $f(k) = A k^{\alpha}$. Then the appendix shows that $k_{t+1} = \alpha \beta A k_t^{\alpha}$ for this case represents an optimal policy

function, while from the national income identity we have

$$C_t = Ak_t^{\alpha} - k_{t+1} = (1 - \alpha\beta)A\, k_t^{\alpha}$$

Given an initial value of k_o, we may actually plot the dynamic predictions for this model, given in figure 1. The first thing to note is that there exists a steady state k equal to

$$\bar{k} = [A\,\alpha\,\beta]^{\frac{1}{1-\alpha}} > 0$$

The capital stock will converge to this steady state over time. Let's assume that we start with a low value of initial capital; i.e. $k_o < \bar{k}$. Then the model predicts that consumption, investment and GNP will all grow monotonically as in figure 1. This is just a simplified version of the neoclassical growth model first developed by Robert Solow in the late 1950's. Countries start with low initial capital stocks. The rate of return on investment is high. At first the growth rate is high as the country accumulates capital at a rapid rate, but then slows down as it becomes wealthier. For some countries (e.g. U.S., U.K.) this is not a bad approximation, at a very broad level. However, we see in section III below how this neoclassical model is in conflict with much data on international growth, and this has recently led to a new strain of growth models.

We mentioned that this model contains falsifiable predictions. What does this mean? In the next section we show how a general equilibrium growth model along these lines, amended for uncertainty, can be evaluated for its ability to explain not only long term trends in growth rates but also the pattern of fluctuations in major macroeconomic aggregates; i.e. business cycles. A major research effort is under way along these lines. It turns out, perhaps not surprisingly, that the simple model above cannot give an adequate explanation of business cycle patterns.

Finally, we should confront an important abstraction that we have made in the expectation of this neoclassical growth model. We have not mentioned "markets" or "prices" anywhere. In fact we employed the fiction of a social planner who chooses everything in this economy; i.e. both consumption and investment.

A justification for this abstraction comes from the first theorem in welfare economics which says that an undistorted market economy is efficient and thus can be represented as a social planning problem; the so-called "invisible-hand" result.[4] Moreover, this optimal social planning outcome can be decentralized through a price system so that producers and consumers will choose the same quantities as in that outcome. We can then feel confident that in describing the dynamics of an economy as a solution to a social planning problem we are actually saying something interesting about the world.

In fact the dynamic programming set-up can be embellished to analyze the institutional details of market decision-making - many of the existing papers in fact take this approach; Mehra and Prescott (1980). We think of the economy as consisting of a large number of individuals who have identical preferences so that there is really a "representative consumer". We may then define equilibrium prices, wages and interest rates, which agents take as given for purposes of decision making. In a competitive equilibrium the values for consumption and investment will be equivalent to those above.

SECTION II : REAL BUSINESS CYCLES

This section reviews the recent attempt to adapt stochastic general

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[4]Lucas and Prescott (1971) first used this in a dynamic context.

equilibrium models to explain macroeconomic fluctuations - i.e. business cycles. Generally known as "real business cycle" (RBC henceforth) theory, this research program is still very new, yet has made significant strides to the extent that it is now one of the major competing paradigms for an understanding of business cycles[5]. This success has not come without a substantial degree of controversy over both its conceptual framework and its methodology, however.

The understanding of business cycles; fluctuations in employment, output, consumption and investment over time, is a central problem in macroeconomics; many would say this defines macroeconomics as a separate field. Up until the mid 1970's, the main framework for understanding business cycles was the Keynesian ISLM model with Phillips curve price dynamics. Many large scale macroeconomic models were based on this structure (and still are to some extent). The "rational expectations revolution" of the 1970's and early 1980's developed by Lucas, Sargent and Barro and others, attempted to explain business cycles as the efficient response of individuals to incomplete information about aggregate demand stocks. These latter generation of models unfortunately turned out to have very little empirical success.

As the development of rational expectations models proceeded however, many technical aspects of the analysis of dynamic, stochastic economic models were worked out. This led researchers to ask the question; Is it possible to construct a dynamic general equilibrium model which can replicate the main features of business cycles in the postwar U.S. economy? A remarkable paper

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[5]An incomplete list of references is Kydland and Prescott (1982, 1988), Long and Plosser (1983), Hansen (1985), Prescott (1986), Barro and King (1985). For a survey see McCallum (1987).

published in 1982, by Finn Kydland of Carnegie Mellon University and Edward Prescott of the University of Minnesota explored this question and argued cautiously in the affirmative.

We will examine the arguments of this paper and some of the many which succeeded it. However, it is beneficial first to provide a brief description of the patterns in the data which these models attempt to describe.

The "business cycle" has come to interpreted statistically as the properties of a detrended time series of macroeconomic aggregates, both prices and quantities. These properties can be described in a number of ways: either in terms of the autocorrelation and cross-correlation properties of different series, or in terms of measures of the overall volatility of the series. Real business cycle models can make predictions for both of these. Evidence from most industrial economies indicates that macroeconomic aggregates tend to move together - there tends to be both positive contemporaneous comovement between output and employment, consumption, and investment. Moreover, there is a considerable degree of persistence over time in movements in each of these series; i.e. each is strongly autocorrelated. For the U.S. economy a 1% increase in output above trend in any year will imply an increase in next years output of between 0.8 and 0.9 per cent. There are important differences, however, in the overall volatility of the different series. Investment fluctuations are greater then output fluctuations (about three times as large), while consumption fluctuates by less than output. Employment, measured as hours worked, fluctuates by about the same amount as output. In addition, these fluctuations in employment over the cycle take place in response to almost noncyclical or slightly procyclical movements in the real wage.

Other features of the business cycle, such as the well known positive relationship between output and price inflation, could be mentioned but to a large extent the importance of such correlations is dictated by the type of model chosen. In the models discussed here, monetary features play no role at all so the inflation output correlation has no importance for the business cycle as defined here. We will return to this issue at the end of the section however.

In the previous section it was shown that a simple general equilibrium growth model could generate a set of time series data for an economy. But some additional structure has to be added to this model to have a serious chance at replicating the data. First of all, it is clear that some stochastic element would have to be added, as the nonstochastic growth model does not display any cyclical phenomena at all. All of the current real business cycle models do this by assuring that the economy's technology is a stochastic process. For given levels of inputs of labour and capital, the final output level fluctuates randomly over time. A second feature we require is to introduce some possibility for variability in employment. The growth model presented above assured that labour supply was fixed at the same level each period; employment did not move at all. In order to get around this the RBC literature amends the growth model by assuming people derive utility from leisure. Thus agents choose how much to work in each period, and aggregate employment can potentially move around over time.

Let us now illustrate an example of an extended growth model that can be solved explicitly for endogenous decision rules for capital, employment and consumption. This environment is also characterized by an equivalence between a social planning problem and a competitive equilibrium so only the former is defined. The techniques of the appendix can be easily

extended to set up a competitive equilibrium for this economy. Take the following problem

(II-1) Maximize $\sum_{t=0}^{\infty} \beta^t U(C_t, \bar{n}-n_t)$

subject to $C_t + I_t = Y_t$

$$Y_t = Z_t f(k_t, n_t)$$

$$I_t = k_{t+1} - (1-\delta) k_t$$

$$\ln Z_t = p \ln Z_{t-1} + \epsilon_t$$

k_o given

This describes a representative consumer who derives utility from consumption C_t and leisure $\bar{n} - n_t$, where $\bar{n}$ is total time endowments and n_t is hours at work. I_t is investment, which equals the time difference in the capital stock plus any depreciation investment, where $0 \leq \delta \leq 1$ is the depreciation rate. Output is a function of current capital and employment, and is affected multiplicatively by a random technology shock Z_t. Assume that $\ln Z_t$ is an AR(1) process with $|p| < 1$, and that ϵ_t is an I.I.D. process with mean zero and variance σ_ϵ^2.

At any time t the consumer observes the current capital stock and technology parameter Z_t, and must decide how much to consume, work and save for the future, given the expected future technology shocks.

This can be set up as a standard dynamic programming problem. The variable k_{t+1} and n_t represent controls, while the state variables are k_t and Z_t. For some restrictions on the functions U and f, we could prove the existence of time invariant decision rules for capital consumption and labour supply. However, in general no analytical solution can be found. But instead let's consider a special case for which exact closed form decision rules may be found. This will provide a natural benchmark against which to compare the more elaborate RBC models discussed below. The special case

involves assuming $U = (1-\theta)\ln C + \theta \ln(\bar{n}-n)$, $f = k^{\alpha} n^{(1-\alpha)}$, and setting $\delta=1$.

With these restrictions the Bellman equation can be written as

$$(II\text{-}2)\quad V(k_t,Z_t) = \max_{C_t,k_{t+1},n_t} \{(1-\theta)\ln C_t + \theta\ln(\bar{n}-n_t) + \beta E_t V(k_{t+1},Z_{t+1})\}$$

$$\text{subject to } C_t + k_{t+1} = Z_t k_t^{\alpha} n_t^{(1-\alpha)}$$

This may be solved using one of the methods described in the appendix. The easiest way is to start "guessing" a form for V and then establishing that the guess is correct. If we guess that $V(k_t,Z_t) = A + B \ln k_t + C \ln Z_t$, then substitute this guess into (II-2), we may solve the maximization problem for the optimal decision rules for C_t, k_{t+1} and n_t. Then substituting the solutions back into the right hand side of (II-2), we have as identity in which the "undetermined coefficients" A, B and C may be obtained. The resulting equilibrium decision rules are given by

$$(II\text{-}3)\quad k_{t+1} = \alpha\beta Z_t k_t^{\alpha}(\bar{n}_t - n_t)^{(1-\alpha)}$$

$$(II\text{-}4)\quad C_{t+1} = (1-\alpha\beta) Z_t k_t^{\alpha}(\bar{n}_t - n_t)^{(1-\alpha)}$$

$$(II\text{-}5)\quad n_t = \frac{(1-\alpha)(1-\theta)\bar{n}}{[(1-\theta)(1-\alpha) + \theta(1-\alpha\beta)]}$$

What do these solutions have to say about business cycles? From (II-3) and (II-4) we see that both investment and consumption are linearly related to output. Thus the positive comovements between various sectors observed in real data is seen as a prediction of this model also. In addition, it can easily be shown that Y_t C_t and k_{t+1} will display persistence over time. Noting that $Y_{t+1} = Z_{t+1} k_{t+1}^{\alpha} n_{t+1}^{(1-\alpha)}$, it follows that $\ln Y_{t+1} = a_o + \ln Z_{t+1} + \alpha \ln Y_t$ where a_o is a constant. Now by (1) $\ln Z_{t+1}(1-pL) = \epsilon_{t+1}$, so we have

$$\ln Y_{t+1} = a_o(1-p) + (p+\alpha)\ln Y_t + p\alpha \ln Y_{t-1} + \epsilon_{t+1}.$$

The log of output thus follows an A R(2) process. This is of some interest

because it is well known that detrended U.S. GNP and other variables may be described quite well by A R(2) processes. In fact both consumption and investment also follow an A R(2) process. It may easily be shown that

$$\ln C_{t+1} = a_1 + (p+\alpha) \ln C_t + p\,\alpha \ln C_{t-1} + \epsilon_{t+1}$$

$$\ln k_{t+1} = a_2 + (p\alpha) \ln k_t + p\,\alpha \ln k_{t-1} + \epsilon_{t+1}$$

In terms of explaining the relative magnitudes of fluctuations, however, the model does very poorly. The stochastic processes governing consumption output and investment are all the same, so their volatility will be the same. This clearly contradicts the known business cycle data. Moreover, from (5) we see that employment is constant and does not fluctuate over time at all. Both these features are implications of the special cases of complete depreciation. If we relax the δ=1 restriction over employment will respond to shocks to Z_t and investment will have a higher variance than either output or consumption. These claims cannot be established analytically however, so it is necessary to look at quantitative analysis of the model.

What is the economic explanation for persistence in this model? The objective of a consumer in this economy faced with a given set of prices and interest rates is to smooth out the flows of consumption over time. Thus individuals behave such as is the permanent income hypothesis of Friedman. A temporarily higher value of Z_t leads to an increase in current income. Individuals wishing to spread this out over time will use part of the higher income to acquire capital, and so future output will be higher, while part will be consumed in the present.

On the face of it, this model suggest that macroeconomic fluctuations over time may just represent the equilibrium responses of consumer/savers in stochastic economy. In order to seriously test the hypothesis that a stochastic general equilibrium model can explain business cycle fluctuations

we have to resort to models that have no explicit analytical solution, and thus optimal decision rules must be obtained through numerical approximations. The quantitative analysis of RBC models began with the paper by Kydland and Prescott (1982). Using a more elaborate version of the stochastic growth model presented above, one in which labour supply can fluctuate in response to shocks along an equilibrium path, they attempted to use this model to mimic the observed data on the U.S. economy. Their model also features exogenous shocks to the technology of production as the main sources of business cycle fluctuations. In equilibrium, agents will try to carry forward positive shocks in the form of higher stocks of capital, thus generating persistent dynamics in output, but they will also respond to high temporary productivity shocks by working harder when the real wage is higher. Thus agents must be willing to substitute the consumption of leisure intertemporally at a fairly high rate - "make hay when the sun shines". This is a vital component of RBC models if they are to succeed in capturing the high variability in hours worked over the cycle, relative to fairly low cyclical fluctuations in the real wage.

Kydland and Prescott elaborated the basic growth model in a number of directions. The novel features of their set-up are a) the inclusion of lagged values of leisure in the utility function to break the strict time separability of preferences which holds in the standard model, b) the presence of "gestation lags" for the formation of new capital, and c) the use of inventories as a separate factor of production. The critical factor in generating realistic results is a). The role of this is to allow for a higher willingness of agents to intertemporally substitute leisure between periods. With utility a function of a weighted sum of current and lagged leisure, if the agent has worked relatively little in preceding periods there is a high

willingness to respond to current increases in opportunities with current work effort - its utility cost is lower. Factors b) and c) are less important and as we see in comparison to subsequent literature, are probably not really necessary.

By far the must novel aspect of the Kydland and Prescott paper, and the feature that has come to be the most controversial in RBC theory, is their approach to empirical testing. It is instructive to quote directly.

> "The test of the theory is whether there are a set of parameters for which the model's comovements ... are quantitatively consistent with the observed behaviour of the corresponding series for the U.S. post-war economy.."

This methodology conflicts sharply with the standard approach of estimation and statistical hypothesis testing in econometric models. It should be fairly clear that any attempt to fit this simple one sector growth model to aggregate macro data is likely to be unsuccessful. We would need a much more disaggregated specification for this. However, if the model was simulated under certain parameter values, and produced time series whose characteristics compare well to the actual time series data, then an argument can be made that the theoretical growth model has explanatory power for business cycles.

The model is "calibrated" by choosing the key parameters of production and technology from a wide variety of outside sources. These sources include panel data on household behaviour as well as secular growth observations. Important parameters are θ, the share attributed to leisure in the utility function, β, the rate of time preference, $(1-\alpha)$, the share of labour in national income, and the degree of intertemporal substitution of consumption and leisure. The advantages of using these estimates from other studies is first that it ties down a number of free parameters in the model, imposing a

natural discipline on the possible simulation results, and secondly most of these parameters have been estimated with a fair degree of confidence, reducing the likelihood of significant measurement error.

How is the comparison between the series made? First of all, the characteristics of various detrended series for the U.S. economy are computed. These are written in terms of the standard deviations of percentage differences trend, as well as the current and lagged correlations with output. Then a similar set of numbers is computed for the model by performing simulations based on the endogenous equilibrium decision rules, and taking averages of the results of these simulations. The model is calibrated so that the standard deviation for output that it implies equals that of the sample data for the U.S. economy for the relevant sample period. This is done by adjusting the distribution of the technology stock Z_t, so that the simulated model arrives at the desired standard deviation for output. As it turns out, it is necessary to have a very high degree of autocorrelation in the exogenous stock to make this work. We return to this below.

Now with two series for the cyclical behaviour of the relevant variable; one for the sample data, and one for the model economy, the test of the models fit is whether the relative ranking of variability of key variables in the simulated model accords with that of the real economy. Perhaps surprisingly, Kydland and Prescott achieve a fair degree of success by this measure.

Results from a number of alternative models are presented in table 1. The first column refers to data for the U.S. economy. The second column refers to figures for a standard neoclassical growth model without the addition of the Kydland-Prescott elaborations. These data are taken from Hansen (1985). We see that this basic model does successfully predict the relative differences in volatility of investment and consumptions observed in

the data, thus represents an improvement over the simple analytical version of the model above. However, the model implies a variability for hours far below the corresponding value for the real economy. This comes from the unwillingness of these theoretical agents to undergo large intertemporal fluctuations in leisure. With the addition of the Kydland-Prescott specification for utility, the variability of hours goes up significantly. Moreover, the ranking of variability, from highest-investment to lowest capital stock, matches that of the economy. Table 2 contains data on the contemporaneous correlation of each variable with output, and again we see here a fairly good fit, (except for the productivity variable, discussed below).

While the variability of hours is increased using the Kydland-Prescott preference structure, the standard deviation of hours is still less than that of the U.S. economy. In addition, the ratio of the standard deviation of hours to that of productivity is higher than that in the data.

Column four of the table reports results of a more recent study by Hansen (1985) which adds on the constraint that hours spent at work are not divisible - people must work either forty hours or not at all. This has the result of considerably increasing the volatility of hours relative to productivity - in fact it makes this ratio even higher than that of the U.S. economy.

Finally column 5 reports more recent estimates from the Kydland and Prescott, reported in a paper by Prescott (1986). The new feature of this paper is that the rate of capital utilization is allowed to vary within a period. Thus technology shocks can lead to increased employment of both labour and capital. This does considerably better than the original Kydland-Prescott model. The total variability of hours is higher, and the

ratio of hours variability to productivity variability is even higher than that for the real economy.

While these statistics are impressive in their ability to match business cycle data, the models have come in for some criticisms. The most important one is the size of the technology shock which is required to have the cyclical behaviour of output in the model correspond to that of U.S. data. Prescott (1986) measures this technology shock through the adaptation of the growth accounting of Solow (1957). The method is to define technological change as the change in total output between periods less the sum of the change in labour times labour's share in output and the change in capital times capital's share in output. This is exactly correct on the assumption that there are only two factors of production. Prescott finds the standard deviation of this change is equal to 1.2 per cent of GNP for the U.S. economy, but concludes that this is biased upwards and revises the estimate down to 0.763 per cent. Furthermore, because the technology variable defined this way, which is just analogous to $Z_t - Z_{t-1}$ in our model above, is approximately white noise, he concludes that the initial technology variable Z_t is highly persistence - almost a random walk, with $p = 0.95$. A standard deviation in technology of this magnitude almost one per cent of GNP, may seem implausibly high. In addition, the degree of persistence is extremely high. One of the problems with the Solow growth accounting methodology is its neglect of other factors of production. If there are significant unmeasured inputs, the residual that is attributed to technology shocks can be drastically reduced.

To some extent the required size of technology variation in these models will depend upon the exact manner in which the technology innovations are introduced into the economy. In the standard Kydland-Prescott economy, an

improvement in technology raises the productivity on all existing capital in the economy (as well as labour). However, it may be more natural to think of technological improvements as being attached to new types of capital available to the firm as distinct from the firm's sunk capital stock. In this case a technology increase gives an incentive for investment in new capital stock and a more rapid depreciation of existing capital. Greenwood, Hercovitz and Huffman (1988) examine a RBC model in which technology shocks are introduced in this way. They find the required persistence in the technology process is much less than that required by the papers above. Intuitively, when a technology shock occurs in their model, a lot more investment will be undertaken to replace historic low productivity capital, generating a lot more persistence from the basic "propagation mechanism" relative to the exogenous shock.

King, Plosser, and Rebelo, (1988) have shown in a recent study of real business cycle models that a high degree of persistence in technology shocks is a necessary requirement to generate persistent movements in output, investment and employment; i.e. to generate business cycle persistence. Barro (1986) likens this to the absence of Keynesian-like multiplier effects in RBC models. To get big fluctuations we need big exogenous shocks. One of the problems attached to this heavy dependence on the technology residual is that it is essentially an unexplained variable. In reality, there is no aggregate technology, and it is difficult to think of examples of breakthroughs in a particular industry's technology which would have an economy wide impact of the magnitude needed for these models. A particularly salient point in this regard is made by McCallum (1987). If there are significant differences among industries then we would expect that technology changes would be independently distributed across industries. When we aggregate across N

industries with equal and independent distributions of technology shocks with variance σ_ε^2 for instance, the aggregate variance is σ_ε^2/N, and thus the economy wide technology variance should be much smaller than that of any particular sector.

Another possibility is that the measured technology shocks are picking up the influence of non-technological supply disturbances such as changes in imported oil prices. If energy is a significant factor of production than this would be the case. Hamilton (1983) finds that significant increases in the price of oil precede every major post-war recession in the U.S. economy except one. Thus accounting for energy price shocks or more generally terms of trade shocks would seem an important next step for RBC theory.

A related type of real shock that may give rise to business cycle effects is suggested by Lilien (1982). He notes for aggregate U.S. unemployment is strongly related, empirically, to the dispersion of unemployment rates across sectors. This suggests an intriguing new possibility for understanding business cycles. Movements in preferences or differences in growth rates of technology between sectors giving rise to required intersectoral movements of resources may give rise to frequent periods of aggregate unemployment if it takes time and resources to move between sectors. The "Lilien hypothesis", as it has come to be called, has been the topic of a lot of empirical and theoretical research. Critics (e.g. Abraham and Katz (1986)) note that by the logic supplied above intersectoral resource flows giving rise to unemployment should also cause increases in vacancies. However, empirically unemployment and vacancies move inversely over the cycle, giving rise to doubts about the hypothesis. Another issue is to identify the types of shocks which are important. Loungani (1986) argues that when we control for oil

shocks in Lilien's model there are no other significant intersectoral shocks that can have business cycle effects.

There is however, a further problem attached to the notion that either technology or unobserved cost shocks drive the business cycle. This is the fact that evidence suggests that the business cycle is not completely a technologically driven phenomena. Take the correlations between productivity and output reported in table 2. For all the simulated models, this is close to unity, reflecting the fact that the Z variable is doing most of the work. However, for the U.S. economy, the correlation is much lower, suggesting the presence of other types of shocks, perhaps demand-driven, which are affecting the movements of output.

This issue is also related to the thorny problem of explaining large cyclical movements in hours in face of relatively non-cyclical movements in the real wage. In the model presented here, the analogue of the real wage is the marginal productivity of labour, and for the Cobb-Douglas production structure this is linearly related to the average product, which is the productivity variable reported in Table 2. If we were to take this model literally as a description of the business cycle, the real wage-output correlation should be a number much higher than anything observed.[6]

A final issue that has generated controversy in the RBC discussion is the method of "calibration" rather than estimation of parameters from the

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[6]In a very recent paper, Christiano and Eichenbaum (1988) calibrate an RBC model allowing for a distribution of shocks to government purchases, in the hope that this would reduce the very high implied correlation between the real wage and employment and output. They find that the correlation is reduced somewhat, but nevertheless remains implausibly high.

data themselves. The problem with the calibration approach is that the final test of the model becomes the subjective one of comparing moments of one sample distribution with those from the model. The RBC literature has no rigorous statistical procedure for deciding when the sample moment is "close" to that of the model. Gregory and Smith (1988) address this problem by noting that one benefit of the general equilibrium model is that the theoretical distribution necessary for hypothesis testing can be generated by Monte Carlo methods from the model itself.

All these models analyze the business cycle in the absence of monetary phenomena. Despite this there is a large amount of evidence on the comovements between nominal and real magnitudes over the cycle - in fact much of the Keynesian macroeconomic tradition attempts to explain this in terms of a fundamental causal link. While not necessarily subscribing to a causal mechanism, it may seem important that the RBC models be able to at least account for these cyclical regularities. King and Plosser (1984) address this question within the context of a RBC framework. In their model, a real equilibrium is completely independent of the stock of nominal money or its growth rate, as is the case by default in the models described above. However, they develop a structure whereby financial intermediaries provide transaction services - in the form of deposit arrangements and cheque-clearing facilities for instance, to firms. The net size of the production of these transaction services can be interpreted as the stock of <u>inside</u> money, created by the private sector. In the presence of a boom in output, driven perhaps by technology improvements, the stock of inside money increases. To the extent that the measured money stock contains a large inside component, the money income relationship observed in the data could imply a significant degree of reverse causation from income to money. King and Plosser provide evidence to

support this argument for the U.S. Independent evidence on this comes from papers by Litterman and Weiss (1986) and Eichenbaum and Singleton (1986). Both these papers contain results which suggest that monetary factors have played very little role in the post-war U.S. business cycle.

This should not be taken as a statement saying that the monetary side of the economy plays no role. Clearly the existence of money is taken as one of the technological features of the real economy, allowing for more efficient trade over a barter economy. Financial institutions may have an enormously important role not only for average output but also for the cyclical properties of output. Indeed we might think of the collapse of credit in the 1930's as a "technological shock" which precipitated the great depression. However, this does not at all establish the case that moderate fluctuations in the quantity of money have an important causal role in business cycle analysis.

SECTION III : ENDOGENOUS GROWTH AND THE BUSINESS CYCLE

The RBC literature summarized above uses the structure of the neoclassical growth model to analyze and understand the patterns of business cycles in a dynamic economy. There are two aspects of the RBC literature and of the neoclassical growth model in general that are important for the discussion of this section. First, in the neoclassical growth model, the determinants of growth in per capita output are not explained in any real sense, but merely assumed by the inclusion of some exogenous technology variable which has a trend growth component. Second, in the RBC models there is essentially no interaction between the factors which determine the cycle and those which determine growth. The empirical analysis emphasized this by detrending the data, thus removing the secular growth component.

At the global level, the neoclassical growth model implies that countries with access to the same technologies should converge to the same rate of growth of per capita magnitudes, even without free international movement of capital. This is patently rejected by data on secular growth across countries. Some low income countries have substantially increased their growth rates and raised living standards, but many have not. Despite the implications of the neoclassical growth model, many economists believe that a country's growth rate is in some way related to domestic factors other than exogenous technical change. This has recently led to an interest in developing models in which the growth rate is endogenous.[7]

A second driving force behind the work on endogenous growth has come from the RBC theory. Since the RBC models focus on technology shocks as the principal component of business cycles, as we have seen, it is natural to think of the combined effect of technology on growth and the cycle. The artificial separation of these two issues in most macroeconomic literature implicitly presumes that the first factor is governed by forces completely independent of the second. We should expect cyclical movements on GNP to have no permanent effects if the growth rate is exogenous to the cycle. However, a lot of recent evidence (e.g. Campbell and Mankiw 1987, Nelson and Plosser 1982) suggests that most shocks to output have <u>permanent</u> effects. This evidence contrasts radically with the reigning paradigm which views the business cycle as random fluctuations about trend, or movements above and below the natural rate. What this evidence suggests is that the natural rate itself is affected by many cyclical shocks.

In order to deal with this weight of evidence against the neoclassical

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[7]See Lucas (1988), Romer (1980), Rebelo (1987).

growth model, RBC theorists have constructed models in which the rate of growth and technology change is endogenous.[8] This allows for a simultaneous combination of cycle and growth. Moreover, by tying the economy's equilibrium growth rate into individual decision-making, it allows for an analysis of the effect of a whole set of exogenous features of the economy (such as policy) on growth.

This approach to integrating cycles and growth is in its infancy so there is very little empirical evidence to speak of as yet. However, it is worthwhile to adapt the standard techniques of dynamic programming once more to examine a model in which there is endogenous and increasing economic growth.

Before we do this however, let's go back to the simple neoclassical growth model of section I. With the sole addition of uncertainty in the manner introduced in section 2, we may write the capital dynamics equation for this economy as

$$k_{t+1} = \alpha \beta Z_t k_t^{\alpha}$$

where Z_t is a random technology shock which we assume has mean 1 and variance 6_Z^2. There are two important aspects of this economy. First of all, it may be shown that the expected growth rate of capital in the steady state is zero. Thus, in expected terms, the model converges to a no-growth equilibrium. Secondly, there exists a unique expected value of the steady state capital stock. This means that temporary shocks on technology; i.e. Z_t shocks, have no long-run effect on the capital stock and hence on anything else in the economy. Thus there is an immediate sense in which Z_t disturbances reflect "cyclical" influences on income as opposed to the "trend" equilibrium of zero

[8] See King and Rebelo (1987), and Romer (1987).

growth rate. While individual shocks to the Z_t variables affect the dynamic adjustment of k_t, they do not affect any of the "long-run" properties of GNP.

This characteristic of the neoclassical model accords well with conventional wisdom on the separation of cycle from growth, but not with the more recent evidence cited above on their interdependence. The basic property comes from the existence of diminishing returns at the aggregate level. An increase in saving in this economy transforms current output into future output at a diminishing rate. This is illustrated in figure 1, for the case of certainty; i.e. $Z = 1$. The S_t curve represents the savings function $S_t = \alpha \beta k_t^{\alpha}$. The concavity of the curve illustrates the property that by increasing the capital shock, we increase savings, but at a diminishing rate. In a steady state with constant capital this must intersect the $k_{t+1} = k_t$ curve. From the shape of the S_t curve it is clear that no matter where the economy starts out, the capital shock will adjust to achieve this steady state.[9]

Now let us take the same model as before but make a small amendment to the technology so that $Y_t = G Z_t k_t$ where G is a constant, and Z_t is the same as before. The key feature of this amendment is to introduce aggregate constant returns to scale in reproducible factors; i.e. we may increase GNP at a nondiminishing rate forever. The dynamic programming representation of the problem to be solved is now

$$V(k_t, Z_t) = \underset{C_t \; k_{t+1}}{\text{Max}} \{\log C_t + \beta EV(k_{t+1} \; Z_{t+1})\}$$

$$\text{subject to } C_t + k_{t+1} = A Z_t k_t$$

The method of solving this is exactly the same as that of section 2. "Guess" that the value function satisfies $V = A + B \ln k_t + C \ln Z_t$. Use this to

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[9]This assumes that we don't start at $k_o = 0$.

figure out the equilibrium decision rules C_t and k_{t+1} conditional on the unknown values of A, B, and C. Then plug the resulting solutions back into the above equation to determine these parameters. Then it is easy to show that the dynamics of the capital stock in this economy is given by

$$k_{t+1} = \beta G Z_t k_t$$

This simple solution has a number of profound implications for thinking about macroeconomics. First of all, notice that the expected rate of growth of capital in the steady state is given by

$$\beta G E(Z) - 1 = \beta G - 1$$

This equals zero only in the special case that $\beta G = 1$. When $\beta G > 1$ then the capital stock grows at a constant expected rate forever. Moreover, this growth rate is determined by the interaction of savings behaviour, β, with the economies technology, G. We have a simple example of an economy where the rate of growth is endogenous. It would be possible to introduce government taxes in this model, for instance an income tax, to show that a higher tax rate reduces the rate of growth of the economy. For the case of certainty, $Z_t = 1$ for all t, then $\beta G > 1$ corresponds to the savings line in figure 2 having a slope greater than unity. Thus there is no stationary level of the capital stock consistent with equilibrium. This is because there is no region of diminishing returns to saving in this economy.

The second important implication that this model has concerns the interaction of business cycles and growth. There is no way in which we can "detrend" the time series in this economy to separate the growth component from the cycle. This is because all shocks to Z_t have a <u>permanent effect on the capital stock.</u> A shock to Z_t has an immediate effect on capital accumulation and output dynamics as in the RBC model. However, it does

permanently change the level of capital stock and output. Thus the same factors which determine the business cycle determine the trend in the "natural rate" in this economy. Thus this very simple example can capture some of the flavour of the empirical findings mentioned above. There is presently a big research effort underway on developing techniques for empirically implementing these models. See King, Plosser, and Rebelo (1988b).

TABLE 1

Standard Deviations of Percentage Differences from Trend

Variable	U.S. Economy	Neo-Classical Model	Kydland-Prescott I	Hansen Model	Kydland-Prescott II
Output	1.76	1.76	1.76	1.76	1.76
Consumption	1.29	0.55	.61	0.51	0.44
Investment	8.6	5.53	6.3	5.71	5.40
Capital Stock	0.63	0.47	.61	0.47	0.46
Hours	1.66	0.7	1.03	1.35	1.21
Productivity	1.18	0.68	0.88	0.5	0.70

TABLE 2

Contemporaneous Correlations with Output (Differences From Trend)

Variable	U.S. Economy	Neo-Classical Model	Kydland Prescott I	Hansen	Kydland Prescott II
Consumption	0.85	0.89	0.94	0.87	0.85
Investment	0.92	0.99	0.80	0.99	0.88
Capital Stock	0.04	0.06	-0.07	0.05	0.02
Hours	0.76	0.98	0.93	0.98	0.95
Productivity	0.42	0.98	0.90	0.87	0.86

SECTION IV : ISSUES IN MACROECONOMIC POLICY

In this section we review some macro policy issues that have been addressed in dynamic models. This is not meant to be an exhaustive list of the possible policy applications of these models, but merely a brief sketch of a number of topics.

One of the important restrictions that the modeling method described above entails is that all policy experiments must be specified in a complete manner. A policy in this context is viewed as a regime or a set of decision rules which determine the actions of the policy maker at any date in any given situation. These actions of course must be consistent with the overall market budget constraint that the government is faced with in making decisions

With these restrictions then, if we wish to determine the effect of any policy change we need to outline the full set of contingency plans associated with the change, ensuring its consistency with the overall budget constraint facing the government. This quite basic requirement has always been recognized in Public Finance theory but until recently has been ignored in many macroeconomic applications.

In dynamic rational expectations models the need to impose such intertemporal consistency requirements is quite obvious. To question the impact of any tax or government spending alteration for instance, we need to ask what this implies for future taxes or spending. Without this, we cannot determine how private expectations respond to the policy change and thus we cannot determine its overall impact.

Three issues are discussed below. The first concerns the effect of government debt and taxes. A simple exposition of the so-called Ricardian Equivalence proposition is presented. The second concerns the

inter-relationship between inflation, money growth, and Fiscal policy as recentiy explored in a number of papers by Sargent and others. Finally, we focus on the concept of 'time consistency' in policy making, which has been the subject of a lot of recent interest.

Public Debt and Taxes

Think of an economy with two sectors. The private sector is populated by a large number of consumers who receive income, consume, borrow or lend, and pay taxes to the government sector. The government sector undertakes direct spending in the economy, financed by a combination of taxes on the private sector and borrowing.

A question of interest in this economy is, holding the entire profile of government spending constant, what is the effect of a switch from tax financing to debt financing (borrowing)? The well publicized Ricardian Equivalence Theorem asserts that under a certain set of conditions there is no effect of this operation on the real economy at all. We provide a simple intuitive example of this result.

Three important conditions must be satisfied for the result: 1) Both sectors have equal access to capital markets at equivalent interest rates, 2) The planning horizon of each sector is the same, 3) The government finances expenditures by nondistorting taxes. Assumptions 1) and 2) are crucial for the sharp distinction between this result and the conventional perspective on the effects of government deficits. This view contends that a switch from taxation to borrowing tends to reduce national saving and either drive up interest rates or push the current account into deficit. Assumption 3) is obviously wrong in a strict sense but to a first approximation it does not make very much difference.

Now let us do a little accounting. Private sector activities in any year can be described by the following balance.

$$A_{t+1} + C_t = Y_t - T_t + (1+r_t)A_t$$

A_t is net asset holdings of the private sector. Assets could be in many forms, e.g. equity loans to foreign countries, or loans to the government. Whatever the form, each asset pays a rate of return of r_t in period t. C_t is private consumption, while Y_t is income. Finally, government levies taxes of the amount T_t which must be paid in period t. The private sector has to choose consumption and next years assets (the control variable), given the state variables Y_t, A_t and T_t. Total private sector savings equals the change in net assets:

$$A_{t+1} - A_t = (Y_t - T_t - C_t) + r_t A_t$$

Now the government sector satisfies the constraint

$$B_{t+1} = G_t - T_t + (1+r_{t+1})B_t$$

B_t is total government liabilities, which pay an interest rate of rt per period.

Government dissaving is now

$$B_{t+1} - B_t = G_t - T_t - r_t B_t$$

Now let us add the private sector's savings to the government savings to set national savings, given by

$$(A_{t+1} - A_t) - (B_{t+1} - B_t) = Y_t - G_t - C_t + r_t(A_t - B_t)$$

Let us think of this as a model of an open economy where the interest rate is fixed in foreign markets. Therefore we want to investigate the effects of a government deficit on the current account. Does higher public borrowing drive the current account into deficit?. Now the current account surplus just equals national savings, so all we have to do is determine the impact of a switch from direct taxation to borrowing on this surplus.

The simple insight that Ricardian equivalence contains is that debt is just deferred taxes. If conditions 1) and 3) hold, the reduction in today's taxes, which is matched by an increase in future taxes, assuming the government satisfies its long-run budget constraint, will leave the anticipated present value of taxes unchanged, and lead to no change at all in private consumption. This means that private savings rise one for one with the reduction in taxes, and hence national savings, or the current account surplus, is unchanged. Thus a pure deficit policy, holding public spending fixed, has no effect on the level of net external debt.

This is shown quite easily. The consumers budget constraints over the whole horizon can be added together to give

$$C_t + E_t\sum_{i=1}^{\infty} C_{t+i}(1/1+r)^i = Y_t + E_t\sum_{i=1}^{\infty} Y_{t+i}(1/1+r)^i - T_t - E_t\sum_{i=1}^{\infty} T_{t+i}(1/1+r)^i$$

$$+A_t(1+r)$$

This just states that the expected present value of consumption must equal the expected present value of disposable income plus original asset holdings. Thus, as far as the consumer is concerned, it is the expected present value of taxes which matter for wealth. However, performing the same operation with the government budget constraint gives us

$$T_t + E_t\sum_{i=1}^{\infty} T_{t+i}(1/1+r)^i = G_t + E_t\sum_{i=1}^{\infty} G_{t+i}(1/1+r)^i + B_t(1+r)$$

This says that the expected present value of tax revenue must equal the expected present value of direct spending plus repayment of current debt. Substituting these two equations together gives

$$C_t + E_t\sum_{i=1}^{\infty} C_{t+i}(1/1+r)^i = Y_t + E_t\sum_{i=1}^{\infty} Y_{t+i}(1/1+r)^i - G_t - E_t\sum_{i=1}^{\infty} G_{t+i}(1/1+r)^i$$

$$+(A_t-B_t)(1+r)$$

Taxes appear nowhere in this equation. Thus, since it is the expected present value of taxes that determine consumer response, not the individual tax rates in any year, and since the expected present value of taxes must go to finance direct government spending, it is only changes in the latter which will affect private behaviour. A deficit which merely postpones taxes leaves their expected present value unchanged and hence has no effect on optimal consumption.

For a closed economy the proposition works symmetrically. In this case national savings are zero, with the real interest rate adjusting to ensure this. What Ricardian equivalence says is that private savings adjust one for one with any reduction in public savings, without necessitating any rise in the real interest rate.

This proposition has generated a lot of controversy to say the least. A large amount of empirical work has not led too a narrowing of viewpoints. The proposition is difficult to test empirically because of the necessary separation between changes in direct government spending and switches in the financing of a given amount of government spending. A recent survey of theoretical work and empirical evidence is given in Bernheim (1987).

One of the obvious criticisms to be made of the result is that if people plan over a life-cycle then the timing of taxes can be important to the extent that switching taxes into the future may reduce their overall tax burden, i.e. they may not be alive to pay the increased taxes. However, in a famous paper, Barro (1974) argued that if savers care about their children, and leave bequests to them, then a current deficit financed by taxes on future generations will merely increase the size of these bequests, leaving current aggregate demand unchanged.

Note that a crucial ingredient of the argument here is that the government does arrange its activities too satisfy a present value budget constraint, which amounts to saying that any (primary) deficit must be matched by a (primary) surplus. (See McCallum, 1984). Descriptive evidence for the U.S. would seem to suggest that the government has incurred an almost continual deficit. However these measures must be corrected for the monetization of deficits and the effect of inflation on the real value of governments' nominal liabilities. (See Barro, 1984.) A recent paper by Hamilton and Flavin (1986) supports the proposition that the U.S. deficit has been balanced in expected present value terms. A similar exercise for Canada has been carried out by Smith and Zin (1988) and supports the same conclusion.

Fiscal Policy and Inflation

Milton Friedman is famous for the pronouncement that inflation is always and everywhere a monetary phenomenon. Ironically, however, some of the recent work in the 'Chicago School' tradition of Friedman emphasizes the importance of fiscal explanations for persistent inflation. In a number of papers, Sargent, and Sargent and Wallace (See Sargent 1985) emphasize the fact that in models encompassing rational expectations, the current inflation rate will be partly determined by expectations of future government policy, and in particular the expectation of future money growth. How is future money growth to be predicted? The key point is that the fiscal and monetary authorities together must satisfy an overall public sector intertemporal budget constraint. A policy of monetary restraint is only credible if it is consistent with the overall budget constraint of the public sector. Sargent and Wallace argued that in face of persistent central government anticipated budget deficits, the private sector will rationally predict that a policy of

low money growth forever is impossible, and will forecast future inflation. Fundamentally, if the public sector budget constraint is to be satisfied, a policy of low inflation forever is impossible in face of persistent and unalterable high budget deficits. Inflation in this sense becomes a 'fiscal phenomenon'. In order to reduce inflationary expectations, and by doing so, have a chance of reducing inflation, the authority must alter the underlying forces which give rise to pressure for monetary growth.

These ideas have been explained with reference to a number of historical anti-inflationary episodes by Sargent (1985). He looks at four cases of hyperinflations in the 1920's, and argues that in all cases the sudden halt to the hyperinflation was more associated with fiscal reform. Each of these countries was experiencing large primary public sector deficits which were being financed almost wholly by monetization. Popular opinion attributes the sudden ending of hyperinflations to currency reform. However, Sargent maintains that previous attempts at currency reform in the absence of fiscal correction had been unsuccessful. Only when the predicted future values of the fiscal deficit were reduced by credible programs to reduce spending and/or increase taxes did the hyperinflations end. In each case cited these endings were dramatic and coincided almost exactly with the fiscal reform.

The key requirement for anti-inflationary policies generally, as Sargent sees it, is to establish the 'credibility' of the reform package. When credibility is low, and it may seem as if inflation is 'entrenched' and removable only with a severe recession. However, if credibility is high inflation can be removed quickly and relatively painlessly. This distinction represents a central difference between rational expectations theories of inflation and the older views based on structural contract rigidities built

into the economy. Sargent argues that the same factors govern the removal of milder rates of inflation. He contrasts the anti-inflation program of Mrs Thatcher in the early 1980's in the U.K., defined as lacking credibility at first, with that of Poincaré in 1920's France, which he argues was immediately successful because it had initial credibility.

Time Inconsistency of Policies

In designing optimal government policies in dynamic models with rational expectations, researchers quickly ran into the following problem. Suppose the authority starting at some date 0, finds it optimal to plan to take action X at some date t>0 in the future. Under rational expectations this announced plan should be expected to guide in some way the current expectations and therefore the actions of the private sector. The question is, will it still remain optimal to take action x at date t? It turns out that in many cases of optimal policy-making it will not. Instead, taking the view that bygones are bygones the authority may find it better from the perspective of time t to diverge from X. In technical terms, this is an example where the principle of optimality mentioned in the first section does not hold. The sequential date by date approach to decision-making does not give us the same answer as the initial overall maximization problem.

This problem arises only when the policy-maker's actions are constrained in some way by an existing distortion in the economy. Examples which have been identified are non-lump sum taxes and the presence of relative price distortions such as might be caused by large trade unions.

The economic implication of this is immediate. If governments cannot precommit their actions then policy is actually made in a sequential 'let bygones be bygones' pattern. Then clearly the original announcement that plan X will be followed has no credibility and thus is not likely to guide

private sector expectations. Instead, expectations will be governed by what people believe what the actual policy will be. Because plan X is the optimal one (by definition) the inability to precommit leads to a lower overall welfare than if precommitment were possible.

This is an example of the 'time inconsistency' phenomenon. The original plan X is not compatible with the incentives that the policy-maker faces along the way, and is thus said to be 'time inconsistent'. This feature of policy problems has generated a lot of interest in the recent macroeconomic literature.

An example may be helpful. Imagine the following very stylized scenario (due to Barro and Gordon 1983a). A monetary authority wishes to have low inflation but high employment. They can directly control inflation but can increase employment only by raising inflation above the rate expected by the private sector. The private sector comprises wage setters who set their rate of wage change equal to their expected rate of inflation. If they forecast inflation correctly then employment is fixed at the 'natural rate' and unaffected by inflation.

What is one authority's optimal choice of inflation rate in this model? Assume that wage setters have rational expectations so that ex post they always predict the correct inflation rate and so inflation has no output effects. Clearly the best thing the authorities can do ex ante is to set the lowest possible inflation rate, say this is zero for our example. This minimizes the welfare costs of inflation given that the authorities have no control over output anyway.

Is this consistent with the incentives the authorities face along the way? To see this, ask what would happen if the wage setters predicted zero inflation, and built this prediction into wage contracts. Then given that

expectations are set, the authority will face an incentive to increase inflation above zero, as long as the perceived benefits in terms of higher output are greater than the direct costs of a higher inflation rate. But if the wage setters have rational expectations they are not likely to predict zero inflation as the choice of the monetary authority in this case. If the wage setters predict inflation correctly, then we end up at an equilibrium where the authorities always choose a positive rate of inflation and the wage setters always predict this rate of inflation. Output is unaffected by inflation, but the direct costs of inflation make this an inferior outcome.

The problem here is the classic 'prisoners dilemma' problem of game theory. The authority would like to guarantee zero inflation, and have people expect this, but this is not possible. if people expect zero inflation, the authority inflates. If people expect positive inflation, then the best thing for the authority to do is inflate. Thus inflation is always a dominant strategy. Zero inflation policy is 'time inconsistent in the sense discussed above.

This phenomenon in the above context and others, has been used to emphasize the dangers of 'discretionary' policy-making as distinct from policy-making governed by rules, which essentially enforce precommitment on policy. In the absence of this time inconsistency phenomenon, it is hard to argue that discretion could be less desirable than rules in policy-making. After all, with freedom of choice we could always choose to replicate the rules policy. However, in cases where precommitment matters, as in the above example, this is no longer true.

A strong criticism of this view of policy is that it takes a very short-run view of the behaviour of the policy-maker. If the 'game' described above is repeated over a number of years, then it may be worthwhile for the policy-makers to build up a reputation by following a low inflation policy for some time. They are maximizing over a longer horizon than suggested above. A number of authors have developed this argument formally, (Barro and Gordon, 1983b, Backus and Driffill, 1985). At a more informal level, casual observation tells us that governments do not renege on their commitments (e.g. in patent law) very often, if at all. This suggests that the empirical content of the type of models which stress 'time inconsistency' is not very large.

CONCLUSIONS

There are many important topics that we have not discussed in this essay. Perhaps one of the most important is the recent work on adapting dynamic general equilibrium models to address issues in international economics and open economy macroeconomics. This work has interpreted the process of external adjustment in an open economy in terms of the underlying dynamics of optimal saving and investment. The original work on this approach was done by Sachs (1981) but there has been a considerable volume of literature since. This literature may be seen in one sense as a natural extension to an open economy setting of the important features of the real business cycle approach, although so far there has been little in the way of empirical results comparable to the closed economy real business cycle models[10].

More generally, in bringing this survey to a conclusion, it seems appropriate to address a number of questions of a broad nature about the importance of the new developments in macroeconomics. First, how has the development of new techniques in the field affected the policy-makers role in the economy? Since macroeconomics is inherently a policy related discipline this must remain one of the principle criteria with which to judge the success or failure of theory. Unfortunately it seems that at the current stage of development in the 'new macroeconomics' it is hard to give a firm answer to this question. Certainly the lesson that policy has to be made within a predictable framework or regime, and the potential dangers of activist monetary and fiscal policies, have infiltrated into the policy arena. In addition it might be argued that the emphasis on 'structural'

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[10] See however, Backus and Kehoe (1988), and Mendoza (1988).

policies such as tax reform reflect in part the new belief in the underlying microeconomic principles behind the macroeconomy. At a more specific level however, the new macroeconomics is not at the stage where it can be used to develop useful empirical models for forecasting and policy evaluation. In order to do this the models would have to be developed to include a much more disaggregated sectoral structure, as well as allowing for a much more realistic role for the government sector. King Plosser and Rebelo (1988b) outline many of the problems of this research agenda.

A second important question is; what should a present day policy-maker conclude from the current line of work about the relevance of the conventional paradigm for the evaluation of policy? Again the question can not be answered with any great precision and certainly not without bringing in the bias of the respondent. Since to a great extent the 'jury is still out' on the results of the new generation of models for policy it is difficult to make recommendations for altering the current policy framework. As we suggested, the main 'negative' lessons of the rational expectations revolution - that unpredictable and erratic policies can be dangerous, seem to have been learned already.

One important aspect of the recent work described in Section III should be noted however. The real business cycle model, like the standard Keynesian and New Classical paradigm before it, clearly separates the determinants of the cycle and those of long term growth. In this sense it remains within the tradition of the Natural Rate theory described by Friedman (1968) - business cycles represent fluctuations around a stable and independent equilibrium growth rate of output and employment. Macroeconomic policy affects only characteristics of the cycle, and not this long term equilibrium. The literature which interlinks cycle and growth represents a sharp break with

this long tradition in macroeconomics. The natural rate is no longer stable and independent of short run phenomena, but may itself be altered in the course of what we call a business cycle. We have also seen that a considerable weight of empirical evidence supports this new way of looking at the data. Policies based on the traditional business cycle perspective may then have to be re-evaluated in the light of these developments.

APPENDIX

This appendix presents a fairly loose description of the technical methods for computing solutions to dynamic programming problems. Consider the problem of maximizing the objective function

(A1) $$\text{Max} \quad \sum_{t=0}^{T} \beta^t r(X_t, U_t) + T(X_{T+1})$$

$$\text{subject to } X_{t+1} = f(X_t, U_t)$$

$$\text{and } X_0 = \bar{X}_o$$

where the variables are given by

a. X_t = state variable

b. U_t = control variable

c. β = time discount factor

The equation f is called a transition equation. It describes the inherent dynamic constraints that agents in this economy are faced with when attempting to determine the motion of the state variable over time.

As explained in the text, we are interested in finding recursive techniques for solving this equation. The result used to obtain this is the Principle of Optimality formulated by Bellman. It says that if we have an optimal solution to problem (A1), then starting at any point along the optimal path, then the continuation of the initial optimal plan must remain optimal. In practical terms this means that we can solve the problem in the following manner. Start at time T, the end of the planning horizon. Given the value of X_T inherited from the past, choose the optimal control to maximize the objective function

$$V_T = r(X_T, U_T) + T(f(X_T, U_T))$$

i.e. choose U_T to maximize benefits for the remaining time, which is just one period, T. This will give us an optimal value $U_T^*(X_T)$, if the r and

T functions obey certain regularity conditions. Thus we can write the solution as $V_T(X_T, U_T^*(X_T)) = V_T^*(X_T)$. Now move back one period and maximize the function

$$V_{T-1} = r(X_{T-1}, U_{T-1}) + \beta V_T^*(f(X_{T-1}, U_{T-1}))$$

In a similar manner this gives an optimal value $U_{T-1}^*(X_{T-1})$ and we have the value of the two period maximum given by $V_{T-1}^*(X_{T-1})$. The process can be repeated back to time period 0. What we are doing in terms of mathematical notation is performing recursions on the equation

$$V_t^*(X_t) = \underset{U_t}{\text{Maximize}}\{r_t(X_t, U_t) + \beta V_{t+1}^*(X_{t+1})\}$$

$$\text{subject to } X_{t+1} = f(X_t, U_t)$$

This is commonly called a 'Bellman equation'. With this we wind up with a sequence of T optimal decision rules. The trick behind dynamic programming is the fact, based on the principle of optimality, that this sequence solves the original decision problem (A1).

In a problem with no natural termination date we cannot use this procedure as a solution method. Such problems are very desirable to focus on however, as they allow for time invariant decision rules. Let us now describe some methods for addressing infinite horizon maximization problems using dynamic programming techniques. It is perhaps simplest to again use the optimal growth example discussed in the text. Thus the problem is

(A2) $$\text{Maximize} \quad \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{subject to } K_{t+1} + C_t = F(K_t)$$

$$K_0 = \bar{K}_o$$

The Bellman equation for this problem is given by

(A3) $$V_t(K_t) = \underset{K_{t+1}}{\text{Maximize}}\{U(F(K_t) - K_{t+1}) + \beta V_{t+1}(K_{t+1})\}$$

Here K_t is the state variable and K_{t+1} is the control variable. There are a

number of mathematical theorems which guarantee that (A3) has a unique, time invariant solution for the function V, called a 'value function', and time invariant decision rules for K_{t+1}, given certain properties of the U and F functions. They need not concern us here. However, we may describe a number of approaches to obtaining a solution. Let us first specialize the problem by assuming that $U(C) = \log C$ and $F(K) = AK^{\alpha}$.

Solution technique 1. Start at any t and use an initial guess for the V_{t+1} function, say for instance $V_{t+1} = 0$. Then repeatedly perform the recursions described by the Bellman equation. For a large number of these recursions, the V function will settle down to its unique, time invariant value. We have as a by-product the decision rules $K_{t+1} = G(K_t)$.

Solution technique 2. Make a 'guess' that the solution to the value function is $V_t = E + B\log K_t$. For our special assumptions it turns out that this guess is correct. To show this substitute the guess into the Bellman equation to get the problem

$$\text{(A4)} \qquad V_t = \underset{K_{t+1}}{\text{Maximize}}\{\log AK_t^{\alpha} - K_{t+1} + \beta(E + B\log K_{t+1})\}$$

If we solve this problem we get

$$K_{t+1} = (\beta B/1+\beta B)AK_t^{\alpha}$$

Now substitute this back into (A4) to get

$$V_t = \log(AK_t^{\alpha}(1/1+\beta B)) + \beta E + \beta B\log AK_t^{\alpha}(\beta B/(1+\beta B)$$

Now equate this solution with our initial guess for V_t and verify that the solution must imply that

$$B = \alpha/(1-\alpha\beta)$$

E can be obtained in a similar way. Substituting the solution for B into the

K_{t+1} solution gives the decision rule for the capital stock used in the text. The rule for consumption can be obtained immediately.

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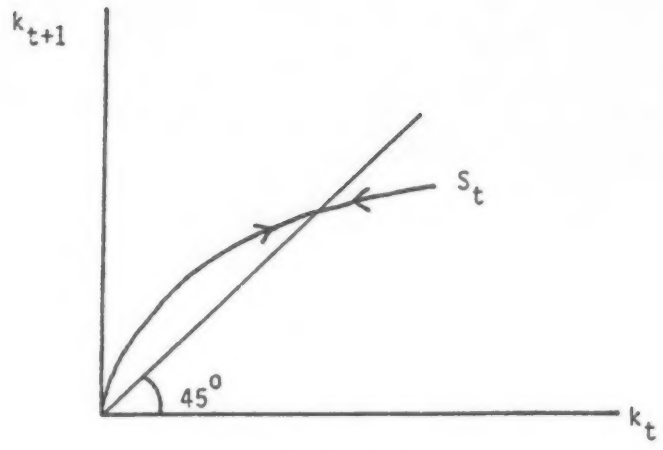

Figure 1

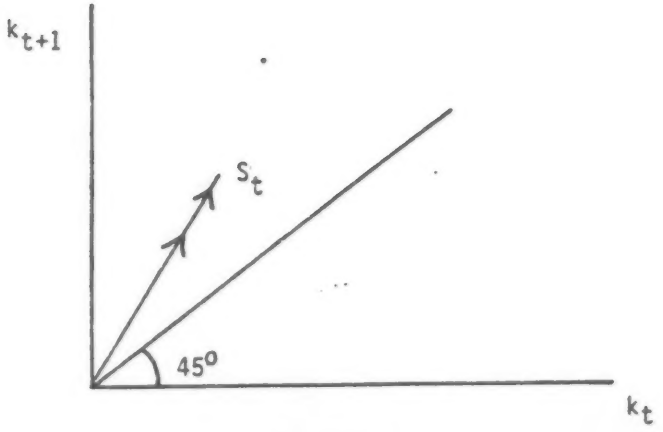

Figure 2